

Effective moment of inertia for several fission reaction systems induced by nucleons, light particles and heavy ions

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Abstract

Compound nucleus effective moment of inertia has been calculated for several fission reaction systems induced by nucleons, light particles, and heavy ions. Determination of this quantity for these systems is based upon the comparison between the experimental data of the fission fragment angular distributions as well as the prediction of the standard saddle-point statistical model (SSPSM). For the systems, the two cases, namely with and without neutron emission corrections were considered. In these calculations, it is assumed that all the neutrons are emitted before reaching the saddle point. It should be noted that the above method for determining of the effective moment of inertia had not been reported until now and this method is used for the first time to determine compound nucleus effective moment of inertia. Hence, our calculations are of particular importance in obtaining this quantity, and have a significant rule in the field of fission physics. Afterwards, our theoretical results have been compared with the data obtained from the rotational liquid drop model as well as the Sierk model, and satisfactory agreements were found. Finally, we have considered the effective moment of inertia of compound nuclei for the systems that formed similar compound nuclei at similar excitation energies.

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The angular distribution of fission fragments is an effective probe to study the dynamics of the fission reaction. The effective moment of inertia of the compound nucleus is a crucial quantity in determining fusion-fission dynamics, although there has not been introduced any precise method to determine it. In this paper, to calculate the effective moment of inertia of the compound nucleus, a novel method is presented. In this method, the values of the effective moment of inertia of the compound nucleus for several induced-fission reaction systems by light projectiles and heavy ions are determined by fission fragment angular distribution method. This method is based upon the comparison between the experimental data of the fission fragment angular distribution as well as the prediction of the standard saddle point statistical model(SSPSM). Calculation through this method is considered with and without neutron emission correction in the reactions. It is now clear that because of the hindrance to fission, a larger number of particles, neutrons in particular, are emitted from the fissioning system. These pre-fission neutrons are emitted not only during the transition stage up to the saddle point but also during the decent from the saddle to the scission point. The emission of neutrons before reaching the saddle point has the effect of lowering the available excitation energy at this point and this in turn will reduce the variance of the K distribution. Assuming the average angular momentum removed as 0.5 per neutron, correction to $\langle I^2 \rangle$ values can also be made. However, the change in the variance of the K distribution due to the angular momentum of emitted neutrons is not significant. In the present work, the pre-fission neutrons are taken to be emitted before the saddle point, since it is not straightforward to separate experimentally the contribution of neutrons emitted before the saddle point and the ones emitted after the saddle point but before the scission point. Afterwards, our theoretical results have been compared with the data obtained from the rotational liquid drop model as well as the rotating finite range model(RFRM), and satisfactory agreements were found. Finally, we have compared the effective moment of inertia for the systems, populating the same compound nucleus and at similar excitation energies.

A study of fission process will produce its own unique insights into the problem of large-scale nuclear dynamics. The study of the fission fragment angular distribution of induced fission by light projectiles and heavy ions continues to be a rich source of information. There is an important physical quantity in the calculation of angular anisotropy of fission fragment by statistical models. This quantity is the effective moment of inertia of the compound nu-

cleus. Fission fragment anisotropy A defined as: $A = \frac{W(0^\circ \text{ or } 180^\circ)}{W(90^\circ)}$. According to the SSPSM, anisotropy A that is proportional to $\langle I^2 \rangle$ is given by: $A \cong 1 + \frac{\langle I^2 \rangle \hbar^2}{4\mathfrak{I}_{eff} T_{sad}}$, where $\langle I^2 \rangle$ is the mean-square angular momentum of the compound nucleus and $\mathfrak{I}_{eff} = \frac{\mathfrak{I}_{||}\mathfrak{I}_{\perp}}{(\mathfrak{I}_{\perp}-\mathfrak{I}_{||})}$ is the effective moment of inertia of the compound nucleus at the saddle point; $\mathfrak{I}_{||}$ and \mathfrak{I}_{\perp} are moments of inertia parallel and perpendicular to the symmetry axis, respectively [1]. T_{sad} , the nuclear temperature at the saddle point can be calculated as: $T_{sad} = \sqrt{\frac{E_{ex}}{a}} = \sqrt{\frac{E_{c.m.}+Q-B_f-E_R-\nu E_n}{a}}$. In this equation, E_{ex} denotes the excitation energy of the compound nucleus, while $E_{c.m.}$, Q , B_f , E_R , ν and E_n represent the center-of-mass energy of the projectile, the Q value, fission barrier height, , rotational energy of the compound nucleus, the number of pre-fission neutrons, and the average energy of an emitted neutron, respectively. The quantity a stands for the level density parameter at the saddle point. Due to the use of the SSPSM, in the calculations, it is assumed that neutrons are emitted before the compound nucleus approaches the saddle point. The average energy carried by an emitted neutron in fission reactions systems induced by nucleons and light particles is assumed to be 5 MeV as well as this energy for each emitted neutron is about 9-10 MeV in induced fissions by heavy ions. The calculation method is based upon the experimental data for the fission fragment angular distribution. It is found that \mathfrak{I}_{eff} is dependent on the second moment of the compound nucleus spin distribution. Afterward, the quantity \mathfrak{I}_{eff} is considered as a linear equation in terms of the center-of-mass energy of the projectile as $\mathfrak{I}_{eff} = a_\nu E_{c.m.} + b_\nu$, where $\nu = 0, 1, 2$ denotes considering without the emission of neutron, emission of one neutron, emission of two neutrons, respectively.

The coefficients of equation \mathfrak{I}_{eff} in terms of $E_{c.m.}$ without considering neutron emission and by considering the correction related to neutron emission for 12 systems undergoing induced fission with nucleons and light particles are given in Table I. The references for finding all necessary data including B_f , $\langle I^2 \rangle$, and the experimental angular distribution data and the bombarding energy range for the systems are also listed in Table I.

In Table I, for some reaction systems, the effective moment of inertia of the compound nucleus is calculated without the emission of neutron, or by taking at most one emitted neutron into account, because the coefficients of equation \mathfrak{I}_{eff} in terms of $E_{c.m.}$ will be negative in terms of emitting two neutrons. It may be noted that the center-of-mass energy of the projectile is roughly the same as that in the laboratory framework for the induced fission performed by light projectiles as well as the rotational energy E_R can be neglected.

The effective moment of inertia for the $p + ^{209}\text{Bi}$ reaction system has also been calculated, but the coefficient of $E_{c.m.}$ in equation \Im_{eff} will be negative considering without the emission of neutron. The effective moment of inertia for this system reaction is similar to the $\tau + ^{207}\text{Pb}$ and $\alpha + ^{206}\text{Pb}$ reaction systems at the same excitation energy for the compound nucleus, since these systems that formed similar compound nucleus, ^{210}Po . In this paper, level density parameter, a is taken $0.094A_{C.N.}$ for all the reaction systems, where $A_{C.N.}$ is the mass number of compound nucleus.

The coefficients of equation \Im_{eff} in terms of $E_{c.m.}$ without considering neutron emission and by considering the correction related to neutron emission for 6 systems undergoing induced fission with heavy ions are given in Table II. The references for finding all necessary data including B_f , $\langle I^2 \rangle$, E_R and the experimental angular distribution data are also given. The bombarding energy range for the systems are also listed in Table II. For all these reaction systems, a is also taken $0.094A_{C.N.}$. The calculated values for \Im_{eff} for the systems undergoing induced fission with nucleons and light particles are compared with those of rotational liquid drop model(RLDM)[11]. It was observed the calculated values to be in agreement with the values obtained by RLDM. In Fig. 1, the effective moment of inertia for the $\alpha + ^{182}\text{W}$ system are compared with those of the RLDM as a typical.

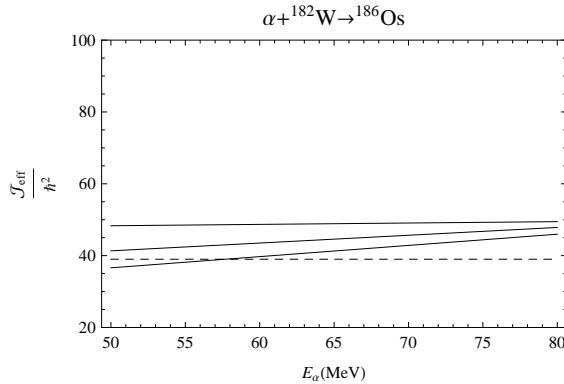


FIG. 1: The comparison of \Im_{eff} obtained from fission fragment angular distribution method with those of RLDM (dashed line) for the $\alpha + ^{182}\text{W} \rightarrow ^{186}\text{Os}$ system. The solid lines from bottom to top represent the effective moment of inertia without correction of neutron emission, with neutron emission correction when the number of emitted neutrons is considered 1 and 2, respectively.

In Fig. 2, the \Im_{eff} for the $^{12}\text{C} + ^{236}\text{U}$ system are compared with those of the rotating finite range model(RFRM) by Sierk[12] as a typical. The calculated effective moment of

inertia in this method was seen to be in agreement with the values obtained by the RFRM.

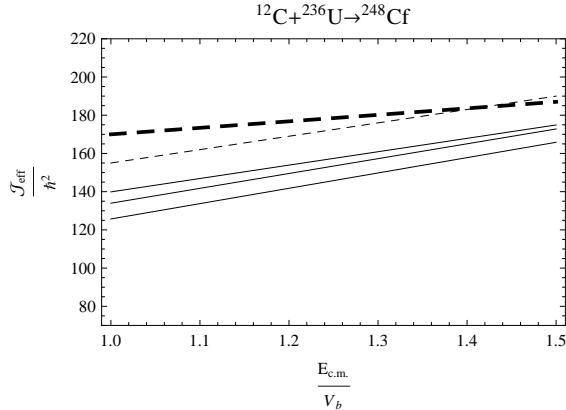


FIG. 2: The comparison of \Im_{eff} obtained from fission fragment angular distribution method with those of RFRM(the long-dashed line) for the $^{12}\text{C} + ^{236}\text{U} \rightarrow ^{248}\text{Cf}$ system. The solid lines from bottom to top represent the effective moment of inertia without correction of neutron emission, with neutron emission correction when the number of emitted neutrons is considered 1 and 2, respectively. The dashed line is the calculated \Im_{eff} for the system by considering correction related to the average number of emitted neutrons[10].

Finally, we have compared the effective moment of inertia for the reaction systems that formed similar compound nuclei. It is expected that this quantity must be the same for the reaction systems populating the same compound nuclei at the same excitation energies. For the $\text{P} + ^{185}\text{Re}$, $\tau + ^{183}\text{W}$, and $\alpha + ^{182}\text{W}$ reaction systems that formed similar compound nucleus ^{186}Os , it is observed that the average errors in the calculation of effective moment of inertia of the compound nucleus ^{186}Os , over the energy range for the reactions mentioned above are $< 5\%$ and $\simeq 29\%$ for the former two reactions in comparison to the last reaction, respectively. Keeping in mind that the effective moment of inertia for the reaction systems populating the same compound nucleus are independent of the role of the entrance channel for these reaction systems and depends on the excitation energy of the compound nucleus. We propose here that the discrepancy between the calculated \Im_{eff} for the $\alpha + ^{182}\text{W}$ reaction system and with those of the $\tau + ^{183}\text{W}$ system arises from the measurement technique of fission fragments(catcher foil technique) which is used in the last experiment[6]. The fission fragment angular distributions are measured using different types of detectors both passive

like mica, lexan and glass and active like gas and silicon surface barrier detectors. A combination of gas and silicon detectors have also been used for a cleaner separation of fission fragments.

We have also compared the effective moment of inertia of the compound nucleus for the reaction systems that formed the similar ^{248}Cf compound nucleus. we have obtained the average errors of the calculated values for the $^{12}\text{C} + ^{236}\text{U}$ and $^{16}\text{O} + ^{232}\text{Th}$ reaction systems, over the energy range for the reactions mentioned above about 6% and < 3% in comparison with those of the $^{11}\text{B} + ^{237}\text{Np}$ reaction system. It has been shown in many instances the the observed anisotropies in fission fragment angular distributions measured in heavy ion induced fission on actinide targets deviate from the predictions of SSPSM. It is generally viewed that this observed fission events consist of an admixture of events of two types: compound nucleus fission(CNF), and non compound nucleus fission(NCNF). The probability of non compound nucleus of fission, $P_{NCNF}(I) = \exp\left[\frac{-0.5B_f(I, K=0)}{T}\right]$, where $B_f(I, K)$ and T are fission barrier height and nuclear temperature, respectively[13]. For the reaction systems where the entrance channel mass asymmetry $\alpha = \left(\frac{A_T - A_P}{A_T + A_P}\right)$ is greater than the Businaro-Gallone(BG) mass asymmetry α_{BG} , the measured anisotropies are in agreement with SSPSM predictions and NCNF is absent. On the other hand, it is observed an anomalous behavior in fission fragment angular distribution for the systems with α , smaller than α_{BG} . While the $^{11}\text{B} + ^{237}\text{Np}$ and $^{12}\text{C} + ^{236}\text{U}$ reaction systems have a normal behavior in fission anisotropies, the contribution of NCNF for the $^{16}\text{O} + ^{232}\text{Th}$ reaction system is less than < 10% over the present energy range of investigation. The effect of this contribution NCNF for the above reaction is the lower average error, that is < 3%.

In conclusion, the calculation of the values of effective moment of inertia using the experimental data for fission fragment angular distribution as well as the prediction of the statistical models is a novel method, which has been carried out in this work for the first time .

In this work, \Im_{eff} is calculated in terms of emitting one and two neutrons and compared with the case of ignoring the neutron emission. It was noted that the values of \Im_{eff} , due to the neutron evaporation, increases, usually less than 10%.

Considering the level density parameter as $a = \frac{A_{C.N.}}{8}, \frac{A_{C.N.}}{10}, \frac{A_{C.N.}}{11}$, rather than $a = 0.094A_{C.N.}$, the quantity for effective moment of inertia of the compound nucleus varies

at the most 6 to 7 %. Hence, this quantity is not sensitive to the level density parameter selected in the computation.

Overall, as experimental values of fission fragment angular distribution are used in this method, we can conclude that fission fragment angular distribution has been successful to calculate the effective moment of inertia of the compound nucleus.

- [1] R. Vandenbosch, and J. R. Huizenga, *Nuclear Fission*(Academic Press, New York, 1973).
- [2] A. V. Ignatyuk, M. G. Itkis, I. A. Kamenev, S. I. Mulgin, V. N. Okolovich, and G. N. Smirenkin, Sov. J. Nucl. Phys. **40**, 400 (1984).
- [3] A. V. Ignatyuk, M. G. Itkis, I. A. Kamenev, S. I. Mulgin, V. N. Okolovich, Yu. B. Ostapenko, and G. N. Smirenkin, Sov. J. Nucl. Phys. **40**, 892 (1984).
- [4] S. D. Beizin, M. G. Itkis, I. A. Kamanev, S. I. Mulgin, V. N. Okolovich, and G. N. Smirenkin, Sov. J. Nucl. Phys. **43**, 883 (1986).
- [5] S. Cohen, and W. J. Swiatecki, Ann. Phys. **22**, 406 (1963).
- [6] A. V. Ignatyuk, M. G. Itkis, V. N. Okolovich, *et al.*, Sov. J. Nucl. Phys. **25**, 13 (1977).
- [7] A. V. Ignatyuk, M. G. Itkis, S. I. Mulgin, *et al.*, JETP Lett., **25**, 58 (1977).
- [8] I. V. Ryzho, M. S. Onegin, G. A. Tutin, J. Blomgren, N. Olsson, A. V. Prokofiev, and P. U. Renberg, Nucl. Phys. A **760**, 19 (2005).
- [9] S. Kailas, Phys. Rep. **284**, 381 (1997).
- [10] S. Soheyli, H. Noshad, and I. Ziaian, IJMPE **17**, No. 2, 425 (2008).
- [11] S. Cohen, F. Plasil, and W. J. Swiatecki, Ann. Phys.(N.Y.) **82**, 557 (1974).
- [12] A. J. Sierk, Phys. Rev. C **33**, 2039 (1986).
- [13] V. S. Ramamurthy, and S. S. Kapoor, Phys. Rev. Lett. **54**, 178 (1985).

fission systems	C.	N.	a_\circ	b_\circ	a_1	b_1	a_2	b_2	$E_{proj.}(MeV)$	References
P + ^{185}Re	^{186}Os	0.362	22.291	0.284	29.890	0.144	41.925	42-66.5	[2, 4]	
$\tau + ^{183}\text{W}$	^{186}Os	0.421	31.853	0.254	43.966	-	68.466	28-59.5	[2, 6, 7]	
$\alpha + ^{182}\text{W}$	^{186}Os	0.312	21.005	0.217	30.483	0.038	46.420	53.5-73	[2, 4]	
$\alpha + ^{185}\text{Re}$	^{189}Ir	0.398	16.573	0.296	26.367	0.087	44.156	46-72.5	[2, 4]	
P + ^{197}Au	^{198}Hg	0.412	20.832	0.327	28.711	0.162	41.883	36-66	[2, 4]	
$\tau + ^{197}\text{Au}$	^{200}Tl	0.366	50.332	0.096	69.203	-	124.503	23-59	[2, 6, 7]	
$\alpha + ^{197}\text{Au}$	^{201}Tl	0.239	33.965	0.105	46.646	-	68.083	47.5-77	[2, 4]	
P + ^{205}Tl	^{206}Pb	0.165	36.319	-	50.080	-	77.087	36-67.5	[2, 4]	
$\tau + ^{207}\text{Pb}$	^{210}Po	0.791	36.373	0.473	57.958	-	-	21-59	[2, 6, 7]	
$\alpha + ^{206}\text{Pb}$	^{210}Po	0.104	39.028	-	53.639	-	80.834	45.5-72.5	[2, 3]	
$\alpha + ^{209}\text{Bi}$	^{213}At	0.276	41.611	0.057	60.694	-	104.841	44-76	[2, 4]	
n + ^{232}Th	^{233}Th	0.478	19.239	0.417	24.934	0.289	34.746	21-95	[2, 5, 8]	

TABLE I: Coefficients of equation \Im_{eff} in terms of $E_{c.m.}$ for light particle induced fission systems without considering neutron emission and by considering correction related to neutron emission.

fission systems	C.	N.	a_o	b_o	a_1	b_1	a_2	b_2	$E_{proj.}(MeV)$	References
$^{12}\text{C} + ^{232}\text{Th}$	^{244}Cm	1.155	55.016	1.033	72.243	0.841	96.353	64-78	[9, 10]	
$^{11}\text{B} + ^{235}\text{U}$	^{246}Bk	10.637	-538.514	11.086	-558.763	11.581	-580.331	60-70	[9, 10]	
$^{14}\text{N} + ^{232}\text{Th}$	^{246}Bk	4.105	-247.861	4.236	-254.123	4.375	-260.208	75-86	[9, 10]	
$^{11}\text{B} + ^{237}\text{Np}$	^{248}Cf	1.910	14.855	1.872	25.527	1.813	39.215	75-114	[9, 10]	
$^{12}\text{C} + ^{236}\text{U}$	^{248}Cf	1.284	45.461	1.243	56.276	1.185	69.816	75-125	[9, 10]	
$^{16}\text{O} + ^{232}\text{Th}$	^{248}Cf	2.087	-54.563	2.072	-45.811	2.042	-34.389	93-136	[9, 10]	

TABLE II: Coefficients of equation \Im_{eff} in terms of $E_{c.m.}$ for heavy ion induced fission systems without considering neutron emission and by considering correction related to neutron emission.